

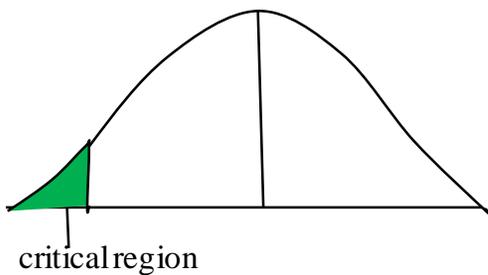
Statistics for Beginners

One-tailed or Two-tailed Tests

In Statistics hypothesis testing, we need to judge whether it is a one-tailed or a two-tailed test so that we can find the critical values in tables such as Standard Normal z Distribution Table and t Distribution Table. And then, by comparing test statistic value with the critical value or whether statistic value falls in the critical region, we make a conclusion either to reject the null hypothesis or to fail to reject the null hypothesis.

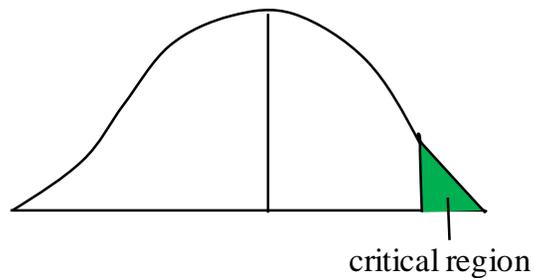
How can we tell whether it is a one-tailed or a two-tailed test? It depends on the original claim in the question. A one-tailed test looks for an “increase” or “decrease” in the parameter whereas a two-tailed test looks for a “change” (could be increase or decrease) in the parameter. Therefore, if we see words such as “increased, greater, larger, improved and so on”, or “decreased, less, smaller and so on” in the original claim of a question ($>$, $<$ are used in H_1), a one-tail test is applied. If words such as “change, the same, different/difference and soon” are used in the claim of the question (\neq is used in H_1), a two-tailed test is applied.

In a one-tailed test, the critical region has just one part (the green area below). It can be a left-tailed test or a right-tailed test. Left-tailed test: The critical region is in the extreme left region (tail) under the curve. Right-tailed test: The critical region is in the extreme right region (tail) under the curve.



Left-tailed test

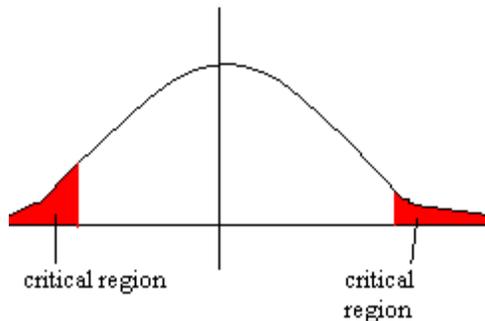
Sign used in H_1 : $<$



Right-tailed test

Sign used in H_1 : $>$

In two-tailed test, the critical region has two parts (the red areas below) which are in the two extreme gerions (tails) under the curve.



Two-tailed test

Sign used in $H_1: \neq$

Example:

*The true value of one type of degree or diploma cannot be quantitatively measured, but we can measure its relative impact on starting salary. Graduates from Quebec universities with a B.A. or B.Sc. degree have a mean annual starting salary of \$28,300. Sixty-five Quebec graduates with a civil engineering degree are randomly selected. Their starting salaries have a mean of \$36,300. If the standard deviation is \$1670, use a 0.01 level of significance to test the claim that Quebec graduates with a civil engineering degree have a mean starting salary that is **greater than** the mean for graduates with a B.A. or B.Sc. degree from Quebec.*

The underlined part of the question above is the original claim of this question. The words "greater than" show that it is a one-tailed (right tail) test and the sign used in H_1 is ">". The null hypothesis and the alternative hypothesis are:

$$H_0: \mu = 28,300$$

$$H_1: \mu > 28,300$$

Note: In MATH 1257 Elementary Statistics, we always use "=" sign for the null hypothesis and the sign of the alternative hypothesis is based on the original claim of the question. In PSYC 2126 & 2127, we need to decide on the symbol used in alternative hypothesis depending on the

original claim of the question first, and then the opposite symbol will be used in null hypothesis. So, the null hypothesis and alternative hypothesis should be:

$$H_0: \mu \leq 28,300$$

$$H_1: \mu > 28,300$$