

Statistics for Beginners

Four Steps of Hypothesis Testing



- **Step one:** State null hypothesis and alternative hypothesis in symbolic form. Usually the hypothesis concerns the value of a population parameter.

How to express null hypothesis and alternative hypothesis in symbolic form

Identify H_1 first. If the original claim of the question uses words such as “greater, larger, increased, improved and so on”, use “ $>$ ” for H_1 . If it uses words such as “less, decreased, smaller and so on”, apply “ $<$ ” for H_1 . If words such as “the same, change, different/difference and so on” appear in the claim, use “ \neq ” for H_1 . The opposite symbol will be used for H_0 . (Note: For MATH 1257, always use “ $=$ ” for H_0 .)

- **Step two:** Compute the test statistics value.
- **Step three:** Identify the critical value or the P-value by the tables.

Be aware of how many tails exist when you look up the critical value in the table. If the symbols “ $>$, $<$, \geq , \leq ” are used in H_1 , it is one-tailed. If the symbol “ \neq ” is used in H_1 , two-tailed.

The significance levels 1%, 5% and 10% are commonly used.

Confidence Level + Significance Level = 1 i.e. Confidence Level = 1 – Significance Level
Therefore, when significance level equals 1%, 5% or 10%, confidence level equals 99%, 95% or 90% respectively. The corresponding critical z values are shown as follows:

Significance Level	Confidence Level	Critical z Value
1%	99%=0.99	2.575
5%	95%=0.95	1.96
10%	90%=0.90	1.645

- **Step four:** Draw a graph included the test statistics value, the critical value and the critical region(s) or compare the P-value with the significance level α . And then make a conclusion of the hypothesis.

Traditional Method: If the test statistics value falls in the critical region(s), reject H_0 . If the test statistics value does not fall in the critical region(s), fail to reject H_0 .

P-value Method: If P-value is less than or equal to the significance level α , reject H_0 . If P-value is greater than the significance level α , fail to reject H_0 .

Example:

The true value of one type of degree or diploma cannot be quantitatively measured, but we can measure its relative impact on starting salary. Graduates from Quebec universities with a B.A. or B.Sc. degree have a mean annual starting salary of \$28,300. Sixty-six Quebec graduates with a civil engineering degree are randomly selected. Their starting salaries have a mean of \$29,100. If the standard deviation is \$1670, use a 0.01 level of significance to test the claim that Quebec graduates with a civil engineering degree have a mean starting salary that is **greater than** the mean for graduates with a B.A. or B.Sc. degree from Quebec.

Solution:

Given information in the question:

$$\mu=28,300$$

$$n=66$$

$$\bar{x}=29,100$$

$$S=1670$$

$$A=0.01$$

Step one: $H_0: \mu=28,300$

$$H_1: \mu > 28,300$$

Step two: parametric \rightarrow one group of samples \rightarrow σ unknown but s is known

Therefore, we use t test with the formula $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$.

Calculate the test statistics t value:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{29100 - 28300}{\frac{1670}{\sqrt{66}}} = 3.89$$

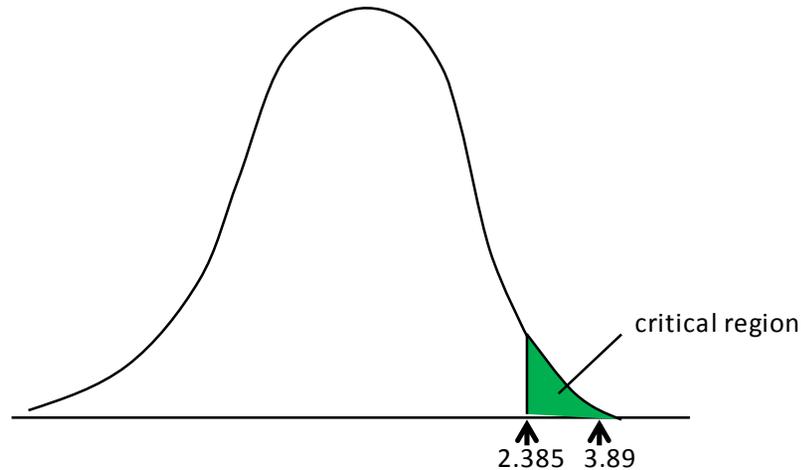
Step three: Identify the critical value or P-value.

We find the critical t value 2.385 by $df=n-1=66-1=65$, $\alpha=0.01$ in t Distribution Table.

Or, we find P-value=0.0001 by using the test statistics $t=3.89$,

one-tailed, in Standard Normal Distribution Table.

Step four: Draw a graph included the test statistics value, the critical value and the critical region(s) or compare the P-value with the significance level α . And then make a conclusion of the hypothesis.



Because the test statistics of $t=3.89$ falls in the critical region, we reject the null hypothesis. Or because the P-value (0.0001) is less than the significance level ($\alpha=0.01$), we reject the null hypothesis. Therefore, we have sufficient evidence to support the claim that that Quebec graduates with a civil engineering degree have a mean starting salary that is greater than the mean for graduates with a B.A. or B.Sc. degree from Quebec. (Note: The null hypothesis and the alternative hypothesis are always opposite, so if we reject the null hypothesis, we accept the alternative hypothesis, i.e. the alternative hypothesis is correct.)